Exercise 25

Show that there are no points (x, y, z) satisfying 2x - 3y + z - 2 = 0 and lying on the line $\mathbf{v} = (2, -2, -1) + t(1, 1, 1)$.

Solution

The parameterization for the line can be written as

$$\mathbf{v} = (2, -2, -1) + t(1, 1, 1)$$
$$= (2, -2, -1) + (t, t, t)$$
$$= (2 + t, -2 + t, -1 + t).$$

The x-, y-, and z-components of the line are

$$x = 2 + t$$
 and $y = -2 + t$ and $z = -1 + t$,

respectively. Substitute these into the equation for the plane.

$$2x - 3y + z - 2 = 2(2+t) - 3(-2+t) + (-1+t) - 2$$
$$= 4 + 2t + 6 - 3t - 1 + t - 2$$
$$= 7 \neq 0$$

Since the right side is 7 for all values of t and not 0, there are no points (x, y, z) on the line that lie in the plane.